The Zelterman estimate of population size under heterogeneity

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Consider a mixture model for a count X of the form $f(x) = \int_0^\infty \exp(-\lambda)\lambda^x / x! d\lambda$ and a sample $x_1, ..., x_N$ arising from this distribution. Due to the sampling mechanism zero-counts are not observed, so that only a truncated sample of size n is observed. It is desired to find an estimate of the population size N. Zelterman (1988 *JSPI*) suggested to use $\hat{N} = \frac{n}{1 - \exp(-\frac{2 f_2}{f_1})}$ where f_j is the frequency of counts equal to j. The Zelterman estimate has been recently utilized to estimate the amount of hidden scrapie in Great Britain (Böhning and Del Rio Vilas 2007 *JABES*). The Zelterman estimate might be understood as a maximum likelihood estimate for a Poisson likelihood with all counts being truncated except x=1 or x=2. This explains the robustness property of the Zelterman estimate since counts larger than 2 will not change the estimate.
Debate is existing whether the Zelterman estimate is a lower or an upper bound for the population size. For the two-component Poisson mixture model this contribution investigates the form of bias associated with the Zelterman estimate. It is shown that asymptotically the Zelterman estimate provides an upper bound for the population size.