# DECOMPOSITION OF SYMMETRY INTO ORDINAL QUASI-SYMMETRY AND MARGINAL EQUIMOMENT AND ORTHOGONALITY OF TEST STATISTIC FOR MULTI-WAY TABLES

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For the analysis of square contingency tables with ordered categories, Agresti (1983) introduced the linear diagonals-parameter symmetry (LDPS) model. Moreover, Agresti (2002) considered the ordinal quasi-symmetry (OQS) model defined by the ordered scores assigned for both the rows and columns. Note that the OQS model with integer scores is identical to the LDPS model. Tomizawa (1991) considered an extended LDPS (ELDPS) model, which has one more parameter than the LDPS model. Agresti described the LDPS model may be appropriate for a square ordinal table if it is reasonable to assume an underlying bivariate normal distribution with equal marginal variances. Tomizawa described that the ELDPS model rather than the LDPS model would be appropriate if it is reasonable to assume an underlying bivariate normal distribution which does not require the equality of marginal variances. These models are special cases of Caussinus (1965) quasi-symmetry (QS) model. Caussinus showed that the symmetry (S) model is equivalent to the QS model and the marginal homogeneity (MH) model holding simultaneously. Tomizawa and Tahata (2007) showed that for multi-way tables the likelihood ratio statistic for the S model is asymptotically equivalent to the sum of those for the QS model of *h*-th order and the marginal symmetry model of *h*-th order. For square tables with ordered categories, Agresti gave a decomposition for the S model into the OQS and MH models. Note that the OQS model is special case of the QS model. Since the OQS model has restrictions stronger than the QS model, we are interested in decomposing the S model into a model with weaker restrictions instead of the MH model.
This paper proposes some decompositions which are different from Caussinus' and Agresti's decompositions. It gives (i) two kinds of decomposition theorems of the S model for two-way tables as follows:

**Theorem 1**
The S model holds if and only if both the LDPS and ME models hold,

**Theorem 2**
The likelihood ratio statistic for the S model is asymptotically equivalent to the sum of those for the LDPS and ME models,

**Theorem 3**
The S model holds if and only if all the ELDPS, ME and VE models hold,

where the ME and VE models are defined by the marginal mean equality and the marginal variance equality respectively.

It also gives (ii) extended models corresponding to the LDPS and ELDPS, and the generalized model further for multi-way tables, and (iii) three kinds of decomposition theorems of the S model into their models and marginal equimoment models for multi-way tables. The proposed decompositions may be useful if it is reasonable to assume the underlying multivariate normal distribution.
Some examples of analysis using biomedical data will be shown.