Almost unbiased estimation of proportions in group testing

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Group testing occurs when units from a population are pooled together and tested as a group for the presence of a particular attribute, such as a disease. If the test is positive, it is assumed that at least one of the units in the group is positive; otherwise, it is assumed that all the units are negative. The purpose of group testing is either to identify the positive units in the groups tested, or to estimate the proportion of positives (p) in the population. The main benefit of group testing is the saving of resources. Our concern here is with estimation, because of the study which gave rise to our work – a survey of virus infection levels in carnations grown in nursery glasshouses. A key feature of that study was a sequential testing procedure, involving three different group sizes.

The MLE of p is biased, and various adjustments have been proposed to correct it, mainly for groups of the same size. We investigate the bias of the MLE when testing groups of different size, using both fixed and sequential procedures. Analytical methods satisfactorily correct the bias for fixed procedures, but for sequential procedures, with their uneven bias patterns, such methods do not work well.

We propose a numerical method of correction which produces an almost unbiased estimator for group testing procedures. It starts with the MLE and progressively modifies it, based on its expectation in the neighbourhood of the estimate. The method is iterative, but even a single iteration produces a substantial reduction in bias. The mean squared error of the proposed estimator is generally comparable to that of the MLE.