Optimal Bandwidth Selection in Reconstruction of Wave Height Fields from Light Transmission Data

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From a wind-wave dynamics study video sequences of slope images in the x- and y- directions (Z_x, Z_y) are collected from which the weighted height field (h) can be constructed [1]. In a discretized form the slope images of dimension mXn, can be expressed in the form of a linear model:

$$Z_y=D_y h + W_x^{-1/2}\varepsilon_x;\ z_y=D_y h + W_y^{-1/2}\varepsilon_y \qquad \textbf{(1)}$$

D_x and D_y are divided difference matrices of dimension (m*n X m*n), W_x and W_y are the weights and ε_x and ε_y are the noises in the x and y directions respectively. The method of regularization is applied for recovery of the wave height image from these data. A weighted penalized objective function corresponding to model (1) has been formulated as

$$l_\lambda(h) = (Z_x-D_x h)' W_x(Z_x-D_x h) + (Z_y-D_y h)' W_y(Z_y-D_y h) + \lambda * h'(P_x' P_x + P_y' P_y + D'_x D'_y D_y D_x + D_y' D_x' D_x D_y)h \qquad \textbf{(2)}$$

where, λ is the regularization parameter and P_x and P_y are Neuman discretization [2].

Optimal reconstruction of height is based on the choice of optimal λ. Although GCV (Generalized Cross Validation) [3] was introduced for its computational simplicity, for a large dimension data Ordinary Cross Validation (OCV) is adopted ahead of GCV as computation of OCV takes a less amount of time compare to GCV due to the sparse matrix computation and odd-even row scheme. The scheme here is that each slope matrix is divided into two sets one with the odd rows (test set) and the other with even rows (validation set). Reconstruction is done on odd rows and validated on even rows. Optimal λ is attained for the minimum value of $OCV(\lambda) = (Z_{xe}-D_x\hat{h}(\lambda)_e)' W_x(Z_{xe}-D_x\hat{h}(\lambda)_e)+(Z_{ye}-D_y\hat{h}(\lambda)_e)' W_y(Z_{ye}-D_y\hat{h}(\lambda)_e)$; e for even rows. Fig1 shows that in a simulated experiment optimal λ is attained at the neighbourhood of minimum MSESlope where MSElope $(\lambda)=(D_x h-D_x\hat{h}(\lambda))' W_x(D_x h-D_x\hat{h}(\lambda))+(D_y h -D_y\hat{h}(\lambda))' W_y(D_y h -D_y\hat{h}(\lambda))$. Finally OCV ($\lambda$) are computed for different values of λ for real data set and height field is reconstructed (Fig2) for the optimal λ.

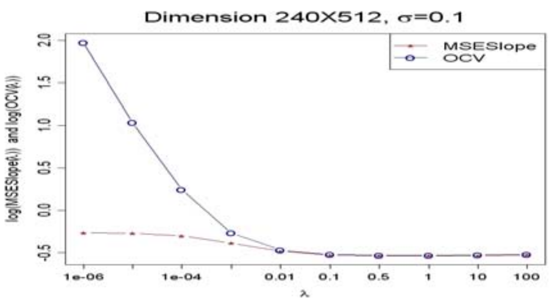

Figure 1: Plot of OCV (λ) and MSESlope (λ) vs. λ and optimal λ= 1 for both cases

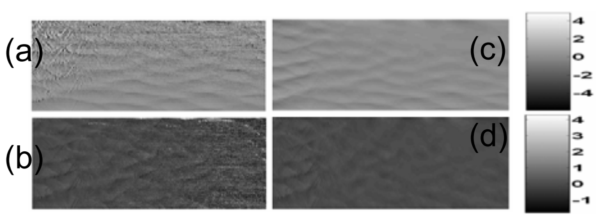

Figure 2: Images of (a) x (b) y-slope data and (c) and (d) corresponding estimated slopes along with (e) the reconstructed height for optimal λ

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[2] O'Sullivan F. Discretized laplacian smoothing by fourier methods, Journal of American Statistical Association: Theory and Methods, 86(415): 634-642, 1991.
[3] Wahba, G. Spline Models in Statisitcs. CBMS-NSF Regional Conference Series, SIAM. PA, 1990