

The exact distribution of the heterogeneity statistic Q in meta-analysis

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We derive the exact distribution of Cochran's Q statistic under the usual one-way normal random effect model in meta-analysis. We then give the corresponding distributions of the recently proposed heterogeneity measures, H^2 , H_M^2 and I^2 , and investigate their properties. The exact power of Cochran's test for heterogeneity is computed, and we derive exact confidence intervals for the heterogeneity parameter when the DerSimonian-Laird or the Hartung-Makambi estimator is used. Two moment-based approximations and a saddlepoint approximation for Q are also developed and compared to the exact distribution in four published applications.