Small optimal designs for a simple logistic model with probability uncertainty

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In many areas of biometry, one considers a process that has a binary response whose probability of success, P, has some uncertainty associated with it. Assume that P is a random variable whose prior is a Beta distribution with mean π. If it is believed that π depends upon a covariate, x, we may postulate that the logit of π is equal to $\eta_x = \beta_0 + \beta_1 x$. We seek an optimal two-point design from which to predict the value of π when only a small number of observations is available. Using a Bayesian approach, the Bayes estimator of π, say $\hat{\pi}$, can be found, but it is biased. In this paper, the Mean Squared Error of $\hat{\pi}$ is used as the basis for an optimality criterion to select a two-point design. We present some examples of optimal designs in terms of η_x.