A Warped Failure Time Model for Human Mortality

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When studying mortality changes over time, traditionally comparisons are based on the hazard function. However, lifespan distributions can also be characterized by their probability density. Instead of modelling trends in the hazards, we may study how the age axis would have to be transformed so that one age-at-death distribution conforms to another. In the simplest case the transformation is linear, leading to a simple accelerated failure time model. A uniform rescaling of the time-axis, however, for the most part is too rigid to capture human mortality dynamics. Therefore more general, that is, nonlinear transformations have to be considered. Ideally we do not want to pick a particular shape of the transformation, but only require it to be smooth.

In "Functional Data Analysis" [5] nonlinear transformation of the axis of the independent variable, to achieve close alignment of functions, is called warping. Hence we call this a Warped Failure Time (WaFT) model. Warping ideas have been introduced into mortality analysis in [2]. In this paper we extend this approach further.

In our model we first choose a target distribution, which for our current application is a Gompertz distribution. The Gompertz distribution plays a prominent role in the study of adult human mortality, but our model is not restricted to this particular target. For an observed age-at-death distribution we seek for the warping function, so that after transforming the age-axis, the density matches the specified target. We assume the warping function to be smooth and represent it by a linear combination of B-splines. The number of knots is chosen purposely high and, following [1], the spline coefficients are restrained by a roughness penalty. The coefficients are estimated via a penalized Poisson likelihood, the penalty controlling the smoothness of the warping function. The B-spline representation naturally allows to incorporate the derivative of the warping function, which is needed in this transformation approach.

We present the estimation algorithm for the WaFT model, demonstrate its performance on simulated data, and then apply it to data from the Human Mortality Database [3]. In particular, we warp standardized age-at-death distributions derived from period life-tables [4]. The resulting warping functions are easy to analyse and allow alternative interpretations of observed mortality development over time.

Generalizations of the WaFT approach to a sequence of warping functions and extensions to non-parametric target distributions are briefly discussed.

References

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