k-FWER Controlling Procedures with Possibly Data-driven Order of Hypotheses

Alessio Farcomeni[1] and Livio Finos[2]

[1] Department of Experimental Medicine, Sapienza University of Rome, Italy
[2] Centro Interdipartimentale di Biostatistica e Bioinformatica, Tor Vergata University of Rome, Italy

Consider the problem of simultaneously testing s null hypotheses. When s is large, the control of the Familywise Error Rate (the probability of falsely rejecting one or more null hypotheses, FWER) may lead to a procedure that is too stringent.
One of the possible solutions is to tolerate more than one false rejection, provided the number of such cases is controlled. Namely, one controls the probability of k or more false rejections, the k-Familywise Error Rate (k-FWER); introduced by Lehmann and Romano (2005).
We focus on cases in which the hypotheses are ordered. This ordering is usually driven by the application, as in dose-response or toxicity studies.
We show that in such cases improvements to the usual k-FWER controlling procedures can be implemented by using a pseudo-gatekeeping approach in which hypotheses are tested in their natural order and rejected until k raw p-values are above the cut-off. We do not rely on any assumption concerning the dependency structure, and show the p-values can come from a parametric as well as a nonparametric setup.
We also show that in the general case the order can be derived by a data-driven criterion which is aimed at increasing power when all variables are deemed to have a similar scale. The data-driven approach can be seen as an extension to k-FWER control of the FWER-controlling procedures proposed by Kropf and Hommel (2004) and Kropf et al. (2004).
We also consider and evaluate how the procedures control the false discovery proportion (FDP), defined by the number of false rejections divided by the total number of rejections.
An application to a real dataset is shown and discussed. R codes are available from the authors.

References

[1] E. L. Lehman and J. P. Romano (2005) Generalization of the Familywise error rate. The Annals of Statistics, 2005, Vol. 33, No. 3, 11381154

[2] Kropf and Hommel (2004) New parametric and nonparametric multiple test procedure for high-dimentional data. Journal of Statistical Planning and Inference Volume 125, Issues 1-2, 31-47. The Third International Conference on Multiple Comparisons

[3] Siegfried Kropf, Jrgen Luter, Markus Eszlinger, Knut Krohn and Ralf Paschke () Nonparametric Multiple Test Procedures with Data-driven Order of Hypotheses and with Weighted Hypotheses