Multiple Comparison Procedures based on Permutation Tests

Aldo Solari[1] and Livio Finos[2]

[1] Department of Chemical Process Engineering, University of Padova, Italy
[2] Centro Interdipartimentale di Biostatistica e Bioinformatica, University of Roma "Tor Vergata", Italy

Permutation-based multiple testing procedures take good advantage of incorporating the dependence structure of the test statistics/p-values to construct valid tests that control the family-wise error rate. For instance, a stepdown procedure such as the Holm's method, which which assumes the least favorable distribution for the p-values, would carry over, *mutatis mutandis*, to the min-P method of Westfall and Young (1993), which estimates the joint distribution of p-values with the hope of greater ability to detect false hypotheses.
The whole subject of multiple testing is frequently, and somewhat innaccurately, called multiple comparisons (MCs), but here by MCs we refer to the comparisons among different treatments. Well-known MC procedures, many of them going back to the 1950s, are reviewed in the book by [2]. However, efficient methods that incorporate the dependence structure of the test statistics rely on parametric assumptions, such as multivariate normality.
MC procedures based on permutation tests has received much less attention. Such issue have been discussed in [3], [4] (see Teresa Neeman's example), and more recently in [5] (see example 6), but the proposed solutions are based on separate permutation distributions of test statistics thus can be computationally prohibitive (don't allow for shortcuts).
In this work multiple comparison procedures based on permutation tests are presented. Our goal is to provide a joint testing family [2] i.e. a joint permutation distribution of test statistics that incorporates their dependence structure.
We discuss the easier setup of testing several treatments against a control and the extension to all pairwise comparisons. The algorithms involved are computationally feasible and permit a general test construction. We illustrate this with some examples. Matlab and R codes are available from the authors.

References

[1] Westfall, P. and Young, S. (1993) Resampling-based multiple testing, Wiley, New York

[2] Hochberg, Y. and Tamhane, A. (1987) Multiple Comparison Procedures, Wiley, New York

[3] Petrondas, D. and Gabriel, K. (1983) Multiple Comparisons by Rerandomization Tests. Journal of the American Statistical Association, 78:949-957.

[4] Westfall, P., Tobias, R., Rom, D., Wolfinger, R., and Hochberg, Y. (1999) Multiple Comparisons and Multiple Tests Using the SAS System, NC: SAS

[5] Romano, J.P. and Wolf, M. (2005) Exact and Approximate Stepdown Methods for Multiple Hypothesis Testing. Journal of the American Statistical Association. 100:94-108.