Analysis of the *lethargic crab disease* by a mathematical model

C. P. Ferreira[1], L. Esteva[2], M. R. Pie[3], P. F. A. Mancera[1], W. A. Boeger[3] and A. Ostrensky[4]

[1] Departamento de Bioestatística, IBB, UNESP, Brazil
[2] Departamento de Matemáticas, UNAM, México
[3] Departamento de Zoologia, GIA, UFPR, Brazil
[4] Departamento de Zootecnia, GIA, UFPR, Brazil

The mangrove crab, *Ucides cordatus*, plays a crucial role in a variety of ecosystem processes in its environment, such as nutrient cycling. Moreover it is an important component in the economy of several underprivileged communities that depend on it for their subsistence. For example, 38% of the households of 21 communities located around the estuary of the Rio Caeté (state of Pará, North Brasil) rely on collection and commercialization of *U. cordatus*. Beginning in 1988, massive mortalities of *U. cordatus*, such as 84% reduction in collection rates, have been report by crab-collectors. Crabs in areas of high mortality share several common symptoms, such as lethargy, poor motor control and inability to return to the upright position when turned upside down. Hence, this pathology is called *lethargic crab disease* (LCD). Several potential etiological agents have been linked in unpublished acconts on LCD, including of exotic metazoans and chemical poisoning. Finally, in 2005, there were several evidences showing that LCD is caused by a fungus of phylum Ascomycota [1]. In this work we developed a mathematical model to describe the LCD disease transmission between the mangrove complexes. The dynamic system is given by the following differential equations:

$$\frac{dS(t)}{dt} = k_1 - \phi\, S(t)\, \left(1 - \frac{S(t)}{C}\right) - \beta(t)\, S(t)\, F(t) - \mu\, S(t) + \gamma\, I(t) \quad (1)$$

$$\frac{dI(t)}{dt} = \beta(t)\, S(t)\, F(t) - (\mu + \alpha + \gamma)\, I(t) \quad (2)$$

$$\frac{dF(t)}{dt} = k + r\, \alpha\, I(t) - \mu_F\, F(t) \quad (3)$$

where $S(t), I(t)$ are, respectively, the susceptible and infected crabs populations, $F(t)$ is the fungus population, k_1 is the migration rate to the susceptible crabs population, ϕ is the birth rate to the susceptible crabs population, C is the carrying capacity, μ is the mortality rate, β is the interaction rate between susceptible crabs population and fungus population, α is the mortality rate to the infected crabs population, γ is a rate of the crabs population which developes resistency, k is the migration rate among estuaries, r is the growth rate to the fungus population and μ_F is the death rate of the fungus population. The equilibrium solutions, the stability analysis, the parametric space and Hopf bifurcations are analised and discussed.

References

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