OVERDISPERSION IN WADLEY'S PROBLEM: THE ALTHAM-POISSON DISTRIBUTION

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Wadley's problem frequently emerges in a dose response context and is one in which the number of organisms that survive treatment with doses of a particular drug is recorded but the number initially exposed to the drug is unknown. This scenario has traditionally been modelled by assuming that the number of survivors, given the number initially treated, follows a binomial distribution and then modelling the binomial parameter n with a Poisson distribution. The resulting distribution is a Poisson distribution with parameter proportional to the probability of success.

The aim of the present study is to accommodate overdispersion in a Wadley problem setting by modelling the response variable with a multiplicative binomial distribution (Altham, 1978) and by placing a Poisson distribution on the unknown number of trials. The resulting Altham-Poisson distribution is investigated in a dose response context. In particular it is shown that the probability of a given response incorporates an infinite sum which can be approximated to any desired degree of accuracy by a finite sum. The estimation of the parameters defining the Poisson-Altham probabilities is also examined and it is noted that, while an elegant EM-algorithm for finding the estimates can be formulated, direct maximization of the log-likelihood is to be preferred. Some comments relating to inference and model checking are included. An example comprising the counts of taken from the paper by Baker, Pierce and Pierce (1980) is used throughout to illustrate the ideas.

References

Altham, PME (1978). Two generalizations of the binomial distribution. Applied Statistics 27, 162-167.

Baker RJ, Pierce CB and Pierce JM (1980). Wadley's problem with controls. GLIM Newsletter 3, 32-35.